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Glacial isostatic adjustment observed using very long baseline interferometry and satellite laser ranging geodesy

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Abstract. In global space geodetic solutions, radial site motions are usually estimated relative to the geocenter (the center of figure of the solid Earth). Most geodesists estimate the motion of the geocenter assuming both that sites do not move radially and that sites move laterally as predicted by plate motion model NUVEL-1A [*DeMets et al.*, 1990, 1994]. Here we estimate the motion of the geocenter assuming that the plate interiors deform radially and laterally as predicted by the postglacial rebound model of *Peltier* [1994] or that of *Peltier* [1996] without assuming a priori knowledge about relative plate motion. Radial site motions estimated relative to this rebound-adjusted geocenter are in the same reference frame as the rebound model predictions, whereas site motions estimated without adjusting for rebound are not. We further constrain the motion of the rebound-adjusted geocenter using satellite laser ranging's sensitivity to the center of mass (of the solid Earth, the oceans, and the atmosphere) by assuming that the mean velocity between the rebound-adjusted geocenter and the center of mass is negligible over the time period of geodetic measurement. Twenty years of observation with satellite laser ranging and very long baseline interferometry record the isostatic response of the solid Earth to the unloading of the late Pleistocene ice sheets. The misfits of the postglacial rebound model of *Peltier* [1994] and that of *Peltier* [1996] are 34% and 16% less, respectively, than the misfit of the rigid plate model. Sites at Onsala (Sweden) and Algonquin Park (Ontario) are observed to be rising at 3 mm/yr and 2 mm/yr, respectively, reflecting unloading of the Fennoscandian and Laurentide ice sheets. Sites along the east coast of the United States are subsiding at < 2 mm/yr, indicating that the forebulge produced by the Laurentide ice sheet is currently collapsing very slowly. Sites beneath the margins of the ice sheets during the last glacial maximum are currently moving laterally away from the ice sheet centers at < 1.5 mm/yr, in disagreement with the moderately fast outward motion predicted by the model of *Peltier* [1996].

1. Introduction

Elevated beach terraces surrounding the Gulf of Bothnia and Hudson Bay record the isostatic response of the solid Earth to unloading of the late Pleistocene ice sheets. Relative sea level histories [e.g., *Pirazzoli*, 1991] determined by radiocarbon dating of these beach terraces and other sea level markers are the main observations constraining viscoelastic models of glacial isostatic adjustment [*Peltier* 1994, 1996] (Plate 1 and Figure 1). Such models account for both the transformation of ice sheets into ocean water and the gravitational effect of changes in the solid Earth on the sea surface. The models depend on two unknown parameters: the mass of the ice sheet as a function of time and the viscosity of the mantle as a function of depth. Using only relative sea level histories to constrain the model leads to highly correlated estimates of the deglaciation history and the mantle viscosity [*Peltier*, 1996]. Errors in the knowledge of either unknown propagates into the inference of the other. Thus the wide range of published values for the mantle viscosity may be a simple consequence of errors in the deglaciation history. Similarly, models of the deglaciation history are sensitive to errors in the radial variation in the mantle viscosity.

The postglacial rebound model of *Peltier* [1994] is determined using relative sea level histories at 414 locations, about half of which were beneath ice sheets during the last glacial maximum 21 ka. Earth is assumed to be laterally invariant and to have the elastic structure of the Preliminary Reference Earth Model (PREM) [*Dziewonski and Anderson*, 1981]. Earth's viscous structure is assumed to have three layers, an elastic lithosphere 120 km thick, an upper mantle and transition zone with a viscosity of 1×10^{21} Pa s above a depth of 660 km, and a lower mantle with a viscosity of 2×10^{21} Pa s (Figure 2). The deglaciation history estimated assuming this Earth structure is ICE-4G [*Peltier*, 1994].

The postglacial rebound model of *Peltier* [1996] is determined from a range of geophysical observations, which allow detailed radial variations in the mantle viscosity to be estimated. The data include the relaxation spectrum for the postglacial rebound of

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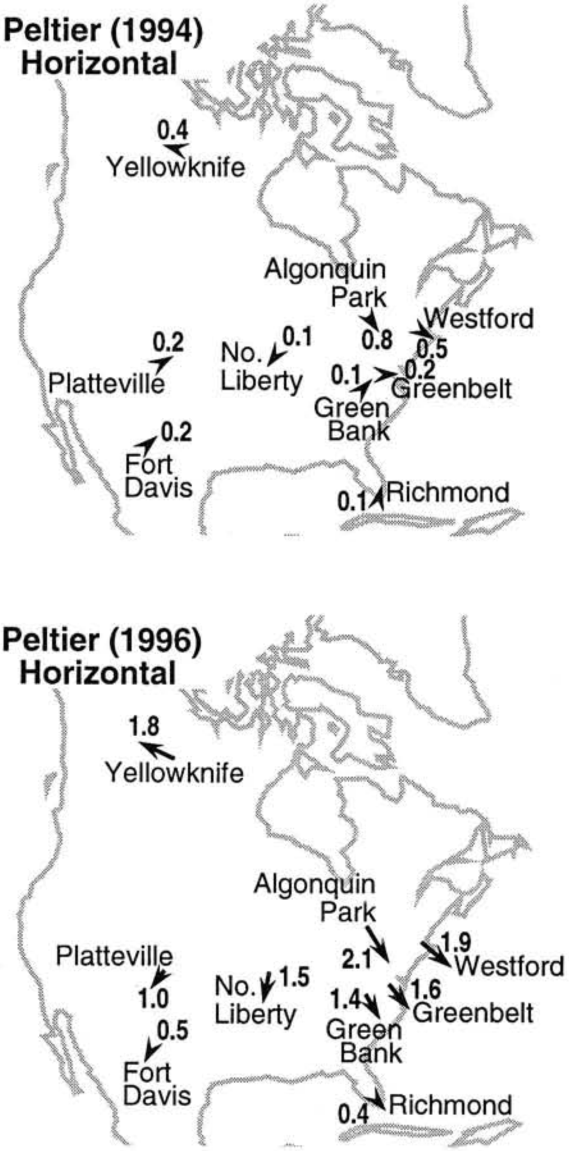


Figure 1. Horizontal motions predicted by the postglacial rebound models of *Peltier* [1994] and *Peltier* [1996]. Areas beneath the Laurentide ice sheet during the last glacial maximum are predicted to move away from the ice sheet center faster in the model of *Peltier* [1996] than in that of *Peltier* [1994]. See *Peltier* [1997] for predicted horizontal motions in Europe.

Fennoscandia [*McConnell*, 1968] and site-specific relaxation times inferred from radiocarbon-dated emergence histories from six sites surrounding the Gulf of Bothnia, seven sites surrounding Hudson Bay, and 10 sites in the Canadian Arctic. The formal inversion is also constrained [*Peltier and Jiang*, 1996] by two anomalies of Earth's present rotational state: the ongoing wander of Earth's (north) axis of rotation at the rate of ~0.95°/Myr along the 76°W meridian [*Vicente and Yumi*, 1969] and the so-called nontidal acceleration of the rate of rotation [*Stephenson and Morrison*, 1995]. The estimated Earth structure has a less viscous upper mantle and a more viscous lowermost mantle than that in *Peltier* [1994] (Figure 2). The ice sheet during the last glacial maximum is, relative to that of *Peltier* [1994], slightly thicker in Fennoscandia and identical in Laurentia.

In this paper we compare the predictions of the postglacial rebound models of *Peltier* [1994] and *Peltier* [1996] with radial and lateral site motions observed using very long baseline interferometry (VLBI) and satellite laser ranging (SLR). The geodetic observations are compared with only these two models because the predictions of no other full postglacial rebound model, consisting of both a deglaciation history and a mantle viscosity profile, are available to us. Furthermore, the mantle viscosity profiles of *Lambeck et al.* [1990], *Forte and Mitrovica* [1996], and *Simons and Hager* [1997] overestimate the characteristic time of exponential decay in uplift of the southeast coast of Hudson Bay [*Peltier*, 1998]. At the heart of our analysis is placing the observations and the predictions into the same reference frame.

2. Reference Frame Definition

Radial site motions may be estimated relative to either of two distinct definitions of the Earth center, the geocenter or the center of mass. The geocenter is the center of figure of the solid Earth. The center of mass of the solid Earth, the atmosphere, and the oceans is the mean point in time about which the satellites rotate.

Defining the reference frame of a global geodetic velocity solution requires specifying the rates of rotation about three normal directions and the rates of translation along three normal directions. When interpreting radial site velocities, the definition of the rates of translation is important [*Heki*, 1996; *Argus*, 1996]. Specifying the velocity of the origin, whether it be the geocenter or the center of mass, is equivalent to defining the three rates of translation. A change in the velocity of the origin in one direction changes all site velocities by an equivalent amount in the opposite direction (Figure 3).

In VLBI geodesy, radio telescopes receive noise signals from quasars distant from Earth. The velocities among the radio telescopes are typically used with various assumptions to infer the velocity of the geocenter. In SLR geodesy, lasers travel between tracking stations and satellites in orbit about Earth. The mean point in time about which the satellites rotate is the center of mass of the solid Earth, the atmosphere, and the oceans. Therefore SLR is sensitive to the center of mass and provides an independent method by which to define the reference frame.

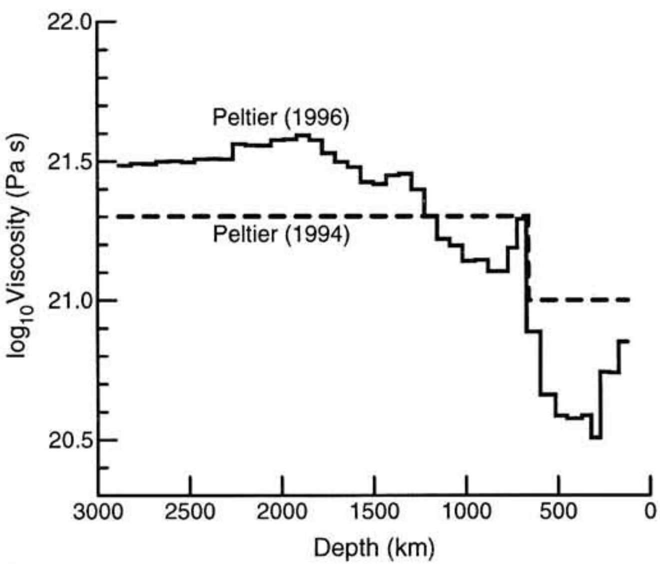


Figure 2. The viscosity of the mantle as a function of depth in the models of *Peltier* [1994] and *Peltier* [1996]. A simple Earth structure is assumed in the model of *Peltier* [1994]; detailed viscosity variations are estimated in the model of *Peltier* [1996].

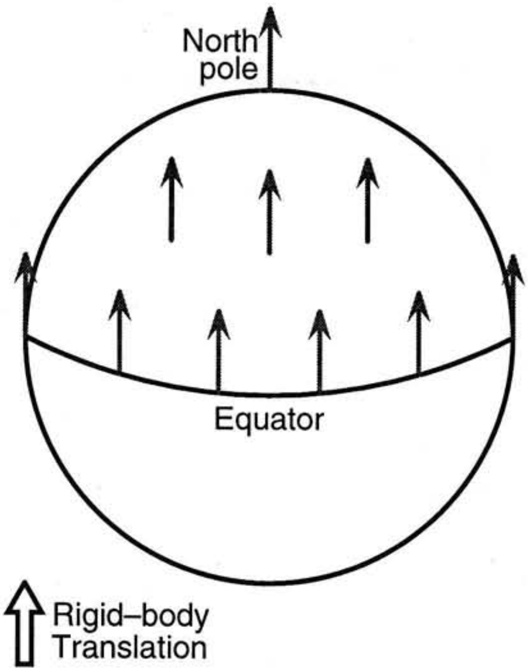


Figure 3. A change in the velocity of the geocenter in the direction of the South pole changes all site velocities by the same amount in the direction of the geocentric vector to the North Pole. The amount by which the radial and lateral components of site velocity change depends on site location. At the North Pole the vertical (uplift) component of site velocity increases. At the equator the north component of site velocity increases.

A range of methods have been used to define the three rates of translation of the VLBI reference frame. *Ma et al.* [solution GLB886a, 1993] and *Ma et al.* [solution GLB907, 1994] define them by fixing the vertical motions of Westford (Massachusetts), Richmond (Florida), and Kauai to zero. Taking advantage of SLR's sensitivity to the center of mass, *Watkins et al.* [1994] place the VLBI solution into the SLR reference frame by imposing velocity ties at sites at the same location. *Heki* [1996] minimizes differences between observed site motions and those predicted by global plate motion model NNR-NUVEL1A [*DeMets et al.*, 1990, 1994; *Argus and Gordon*, 1991]. C. Ma and J. W. Ryan (solution GLB1014j, electronic communication, 1996) minimize differences between observed lateral site motions and NNR-NUVEL1A predictions. *Argus* [1996] minimizes differences between observed radial site motions and those predicted by the postglacial rebound model of *Peltier* [1994]. The different methods result in considerably different site motion estimates: the velocity of the origin differs by 1.6 mm/yr between *Watkins et al.* and *Ma et al.* [1993], by 3.7 mm/yr between *Heki* [1996] and *Ma et al.* [1994], and by 1.7 mm/yr between *Argus* [1996] and C. Ma and J. W. Ryan (solution GLB1014j, electronic communication, 1996). We next assess the methods by which to define the reference frame.

2.1. Rebound-Adjusted Geocenter (RAG)

Because the geodetic sites sample the surface sparsely, they alone cannot be used to determine the location of the geocenter. If, however, the sites are assumed to move tangent to the surface, the velocity of the geocenter can be estimated. The radial motions due to postglacial rebound violate the assumption that there are none.

Radial site motions predicted by postglacial rebound models are described relative to the geocenter in the equilibrium state of the solid Earth, that is, in the state of the solid Earth undeformed by loading of the ice sheets. To place the geodetic observations into the same reference frame as the model predictions, we estimate radial site velocities relative to this rebound-adjusted geocenter (Figure 4). The lateral and radial predictions of the postglacial rebound model of *Peltier* [1994] or that of *Peltier* [1996] are first subtracted from the estimated site velocities. The velocity of the rebound-adjusted geocenter and the angular velocities of the plates that minimize the sum of squares of differences between the adjusted observations and the plate model predictions are next estimated. For the radial component of site velocity, the model prediction is the negative of the projection of the velocity of the geocenter onto the local vertical direction at the site. For the lateral components of site velocity, the model prediction is the sum of (1) the negative of the projection of the velocity of the geocenter onto the local horizontal plane at the site, and (2) the plate velocity, which equals the cross product of

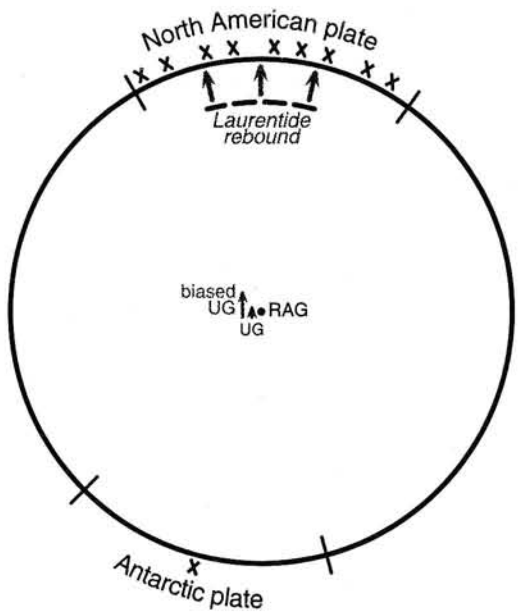


Figure 4. Schematic diagram illustrating the motion between the rebound-adjusted geocenter (RAG), the unadjusted geocenter (UG), and the apparent unadjusted geocenter (biased UG) estimated from the observed site motions. There are many sites (crosses) on the North American plate and one site on the Antarctic plate. Laurentia is rising from its current deformed state (dashed) toward its equilibrium state (solid). The motion between the geocenter and rebound-adjusted geocenter is in reality negligible, ~0.03 mm/yr. This real motion is calculated by approximating the rebound of Laurentia as a disc 1100 km wide rising at 10 mm/yr. However, the motion of the unadjusted geocenter estimated from the observed site motions is probably biased because the component of this motion in the direction of Laurentia depends primarily on the radial motions of sites in Laurentia, a significant fraction of which are affected by postglacial rebound. The motions of sites not on either the North American or Antarctic plates are poor measures of the motion of the geocenter in the direction of Laurentia because it is difficult to determine how much of the site motion is associated with plate motion. Setting plate motion equal to that in model NUVEL-1A [*DeMets et al.*, 1990, 1994] may result in biases due both to inaccuracy in the plate motion model and to differences between plate motions over the past several years and those over the past 3 Myr.

the angular velocity of the plate and the geocentric vector to the site. Implicit in this formulation is the assumption that the plates rise, fall, and otherwise deform as predicted by the rebound model for which the observations are adjusted.

The vertical observations constrain the velocity of the geocenter because if the estimated velocity of the geocenter is wrong, a plate on one side of Earth will appear to rise (or fall) while the plate on the opposite side of Earth will appear to fall (or rise). The horizontal observations constrain the velocity of the geocenter because changing the velocity of the geocenter changes the horizontal component of site velocities by different amounts at different locations (Figure 3) and, if the estimated velocity of the geocenter is wrong, the plates will appear to deform laterally.

2.2. Center of Mass (CM)

In SLR solutions, the center of mass of the solid earth, the oceans, and the atmosphere usually defines the three rates of translation of the reference frame. For example, radial site motions in solution SLR CSR96L01 [R. J. Eanes and M. M. Watkins, electronic communication, 1996) are described relative to the center of mass. The motion of the geocenter need not be estimated. If it is, the inversion provides an estimate of the mean velocity between the geocenter and the center of mass over the time period of observation.

2.3. Rebound-Adjusted Geocenter Equals Center of Mass (RAG=CM)

No phenomenon is known to sustain for decades a significant velocity between the center of mass and the geocenter (or the rebound-adjusted geocenter). Postglacial rebound, the eustatic rise of sea level, ice sheet changes, continental drift, mantle convection, and construction by man redistribute mass but appear to produce < 0.1 mm/yr of motion between the center of mass and the geocenter. Rough calculations of the geocenter motion produced by these phenomena were included in the submitted version of this manuscript and are available from D. F. Argus. Apparent seasonal fluctuations between the center of mass and geocenter observed using SLR have peak-to-peak amplitudes of ~40 mm, but such fluctuations average to a velocity insignificantly different from zero [*Watkins and Eanes*, 1993, 1997; *Kar*, 1997]. Seasonal fluctuations caused by observed variations in the atmosphere, the oceans, and the Earth's groundwater have peak-to-peak amplitudes of ~5 mm and average to a velocity of less than 1 mm/yr [*Dong et al.*, 1997]. The current offset between the center of mass and the geocenter can be calculated using the spherical harmonic coefficients of *Pavlis and Rapp* [1990]: the geocenter is 1.2 km nearer the surface location 46°N, 34°E than the center of mass. This small offset, which is 0.02% of Earth's radius, suggests that the velocity of the geocenter relative to the center of mass may be insignificant. It seems reasonable to assume that the velocity between the center of mass and the geocenter is negligible over the 20-year period of geodetic observation.

2.4. Unadjusted Geocenter Assuming a Plate Motion Model (UG–NUVEL1A)

Most geodesists define the three rates of translation of the reference frame by assuming that sites do not move radially and that sites move laterally as predicted by no-net-rotation plate motion model NNR-NUVEL1A [*DeMets et al.*, 1990, 1994; *Argus et al.*, 1991]. For example, the reference frame for the latest VLBI solutions (GLB1014j, GLB1083c, C. Ma and J. W. Ryan, electronic communication, 1996, 1997) is defined by minimizing differences between observed lateral site motions and NNR-NUVEL1A predictions. Similarly, the reference frame for the 1994 and 1996 International Terrestrial Reference Frame (ITRF) velocity solution [*Boucher et al.*, 1996, 1998] is defined by minimizing differences between observed radial and lateral site motions and NNR-NUVEL1A predictions. (The 1996 realization of the ITRF is the reference frame in which geoscientists determine site motions using the Global Positioning System (GPS).) Such reference frame definitions are subject to errors in NNR-NUVEL1A and to differences between plate motions over the past several years and those over the past 3 Myr.

3. Data Reduction

3.1. Data

We combined VLBI solution GLB1083c (C. Ma and J. W. Ryan, electronic communication, 1997) and SLR solution CSR96L01 (R. J. Eanes and M. M. Watkins, electronic communication, 1996). VLBI solution GLB1083c is determined from interferometric data taken from November 1979 to July 1997. Of the solution's 82 site velocities, 75 meet the criteria that the velocity be estimated from four or more data in three or more calendar years over a time period of 2 years or longer. The velocity estimates are of high quality. The horizontal formal standard (1σ) errors are less than 1 mm/yr at 65 sites, less than 2 mm/yr at seven more sites, and greater than 2 mm/yr at three sites. The vertical formal standard errors are less than 1 mm/yr at 39 sites, less than 2 mm/yr at six more sites, and greater than 2 mm/yr at 30 sites. The median time span of observations is 6 years; 23 sites have data over 8 years or longer.

SLR solution CSR96L01 is determined from laser ranging data taken from May 1976 to February 1996. Of the solution's 72 site velocities, 43 meet the criteria that the horizontal formal standard errors be less than 7 mm/yr. The velocity estimates are of variable quality. The horizontal formal standard errors are less than 1 mm/yr at 11 sites, less than 2 mm/yr at nine more sites, and greater than 2 mm/yr at 23 sites. The vertical formal standard errors are less than 1 mm/yr at nine sites, less than 2 mm/yr at three more sites, and greater than 2 mm/yr at 31 sites. The median time span of observations is 8 years; 26 sites have data over 8 years or longer. Three estimated vertical site motions are anomalous: -5.2 ± 1.1 mm/yr at Maui, -9.0 ± 3.1 mm/yr at Orroral (Australia), and -28 ± 7 mm/yr at Haystack (Massachusetts). (Hereinafter the values quoted after the "±" are 95% confidence limits.) These three anomalous vertical site motions were treated as outliers, that is, they were omitted. Two of the three vertical site rates are suspect for independent reasons: Haystack's huge subsidence is estimated from only two data taken 13 years apart, and Maui's velocity is estimated differently from all other velocities in that biases as a function of range were estimated.

3.2. Assignment of Sites to Plates

We assigned 29 VLBI and 14 SLR sites to one of eight plates (Table 1). Sites were assigned to plates on the basis of the distribution of Holocene faulting, large and great historical earthquakes, seismicity, and topographic relief following *Argus and Gordon* [1996]. The western limit of the North American plate was taken to be the boundary between the Great Plains and Rocky Mountains. The Colorado Plateau was not assumed to be part of the North American plate because the two are separated by the Rio Grande rift. The VLBI site at Pietown (New Mexico) is observed to move relative to the stable North American plate interior toward the west at a marginally significant rate of 1.4 ± 1.3

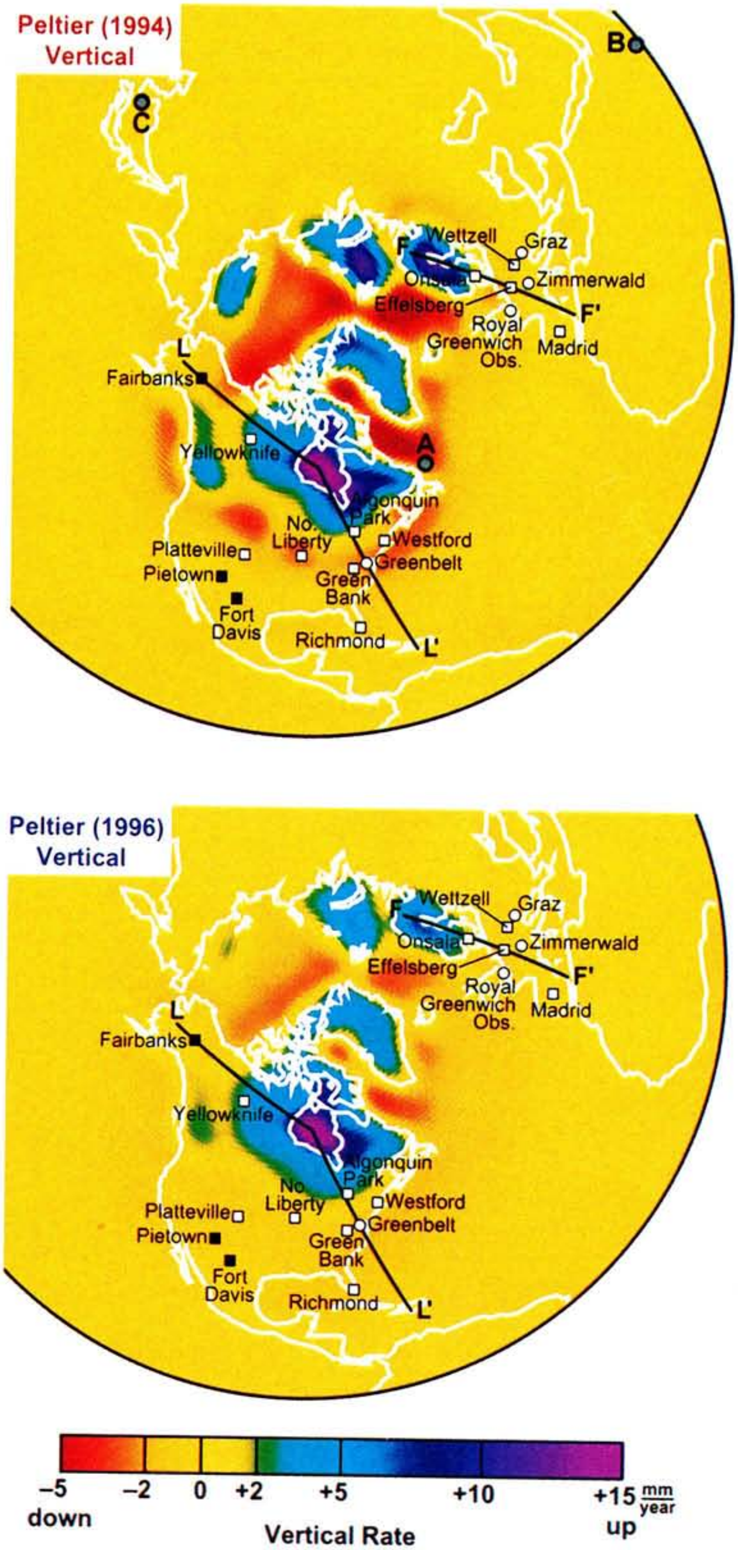


Plate 1. Vertical motions predicted by the postglacial rebound models of *Peltier* [1994] and *Peltier* [1996]. The locations of VLBI (squares) and SLR (circles) sites on plate interiors (white symbols) and along plate margins (black symbols) are shown. There are local maxima in uplift in Hudson Bay and the Gulf of Bothia, at the centers of the late Pleistocene Laurentide and Fennoscandian ice sheets, respectively. The collapse of the forebulge produced by the Laurentide ice sheet is predicted to be faster in the model of *Peltier* [1994] than in that of *Peltier* [1996]. Vertical motions along profiles across Laurentia (L-L') and Fennoscandia (F-F') are plotted in Figure 6. Geocenter vectors to surface locations A, B, and C define the coordinate system in Plates 2 and 3 and are parallel to the minor, intermediate, and major axes, respectively, of the error ellipsoid describing the uncertainty in the motion of the rebound-adjusted geocenter relative to the VLBI network estimated without using SLR's sensitivity to the center of mass.

Table 1. Site Locations, Plate Assignments, and Data Importances

Site Abbreviation	Technique	Latitude °N	Longitude °E	Horizontal ι	Vertical ι	Site Location
North American plate						
GRF105	SLR	39.02	−76.83	0.77	0	Greenbelt, Maryland
YLOW7296	VLBI	62.48	−114.47	0.42	0.01	Yellowknife, Northwest Terr.
WESTFORD	VLBI	42.61	−71.49	0.39	0.10	Westford, Massachusetts
RICHMOND	VLBI	25.61	−80.38	0.37	0.03	Richmond, Florida
PLATTVIL	SLR	40.18	−104.73	0.32	0	Platteville, Colorado
NRAO 140	VLBI	38.44	−79.84	0.30	0.08	Green Bank, West Virginia
HAYSTACK	VLBI	42.62	−71.49	0.27	0.06	Haystack, Massachusetts
ALGOPARK	VLBI	45.96	−78.07	0.26	0.06	Algonquin Park, Ontario
PLATTVIL	VLBI	40.18	−104.73	0.28	0.01	Platteville, Colorado
HAYSTK	SLR	42.62	−71.49	0.24	omit	Haystack, Massachusetts
RICHMO	SLR	25.61	−80.38	0.22	0	Richmond, Florida
MARPOINT	VLBI	38.37	−77.23	0.12	0.01	Maryland Point, Maryland
NRAO85 3	VLBI	38.43	−79.84	0.12	0.01	Green Bank, West Virginia
NL-VLBA	VLBI	41.77	−91.57	0.12	0.01	North Liberty, Iowa
HN-VLBA	VLBI	42.93	−71.99	0.10	0.01	Hancock Park, New Hampshire
GGAO7108	VLBI	38.83	−76.83	0.10	0.01	Greenbelt, Maryland
NRAO20	VLBI	38.25	−79.83	0.07	0.00	Green Bank, West Virginia
Eurasian plate						
ONSALA60	VLBI	57.40	11.93	0.85	0.14	Onsala, Sweden
DSS65	VLBI	40.43	−4.25	0.63	0.03	Madrid, Spain
WETTZELL	VLBI	49.15	12.88	0.55	0.10	Wettzell, Germany
EFLSBERG	VLBI	50.52	6.88	0.53	0.10	Effelsberg, Germany
RGO	SLR	50.87	0.34	0.49	0	Royal Greenwich Obs., England
GRAZ	SLR	47.07	15.49	0.49	0	Graz, Austria
ZIMMER	SLR	46.88	7.47	0.28	0	Zimmerwald, Switzerland
WETZEL	SLR	49.15	12.88	0.26	0	Wettzell, Germany
NYALES20	VLBI	78.86	11.87	0.23	0.00	Ny Alesund, Spitsbergen Isl.
POTSDM	SLR	52.38	13.06	0.15	0	Potsdam, Germany
Australian plate						
YARAG	SLR	−29.05	115.35	1.44	0	Yaragadee, Western Australia
ORRLLR	SLR	−35.64	148.94	1.07	omit	Orroral, New South Wales
DSS45	VLBI	−35.40	148.98	0.89	0.03	Tidbinbilla, New South Wales
HOBART26	VLBI	−42.80	147.44	0.84	0.02	Hobart, Tasmania
PARKES	VLBI	−32.82	148.26	0.12	0.00	Parkes, New South Wales
Pacific plate						
HOLLAS	SLR	20.71	−156.26	1.04	omit	Maui
KAUAI	VLBI	22.13	−159.67	0.89	0.05	Kauai
MARCUS	VLBI	24.29	153.98	0.56	0.00	Marcus Isl.
KWAJAL26	VLBI	9.40	167.48	0.44	0.00	Kwajalein
HUAHI2	SLR	−16.73	−151.04	0.40	0	Huahine
KOKEE	VLBI	22.13	−159.67	0.25	0.01	Kauai
MK-VLBA	VLBI	19.80	−155.46	0.22	0.01	Mauna Kea
African plate						
HARTRAO	VLBI	−25.89	27.69	2	0.06	Hartebeesthoek, South Africa
Antarctic plate						
OHIGGINS	VLBI	−63.32	−57.90	2	0.01	O'Higgins, Antarctica
South American plate						
FORTLEZA	VLBI	−3.88	−38.43	2	0.01	Fortaleza, Brazil
Nazca plate						
EASTR2	SLR	−27.15	−109.38	2	0	Easter Isl.
SLR Subtotal				9.14	0	
VLBI Subtotal				15.88	0.98	
Total				25.02	0.98	

The importance (ι) of a datum is a measure of what fraction of a parameter the datum is constraining [*Minster et al.*, 1974]. The importance of the vertical and horizontal site velocity components are listed for the model in which the velocity between the rebound-adjusted geocenter and the center of mass is assumed to be negligible over the time period of observation. The data importances sum to the number of estimated parameters, 26. The three rates of translation of the SLR reference frame are fixed to zero; therefore the vertical SLR data have zero importance. The three rates of translation of the VLBI reference frame are constrained by VLBI vertical data ($\iota = 0.98 = 33\%$ of 3) and by VLBI horizontal data ($\iota = 2.02 = 67\%$ of 3). The three rates of rotation of the SLR reference frame are constrained by only SLR horizontal data ($\iota = 3$). The three rates of rotation of the VLBI reference frame are constrained by only VLBI horizontal data ($\iota = 3$). The plate angular velocities are constrained by SLR horizontal data ($\iota = 9.14 - 3 = 6.14 = 36\%$ of 17) and by VLBI horizontal data ($\iota = 15.88 - 2.02 - 3 = 10.86 = 64\%$ of 17).

Table 2. Models Defining the Reference Frame in Different Ways

Model, NSSD	Input Data	Fixed Parameters	Estimated Parameters	Assumptions
CM, 1.04*	14 VLBI site motions, 12 SLR site motions, minus 2 anomalous SLR radial motions. 14×3 + 12×3 − 2×1 = 76 data.	SLR frame translation, SLR frame rotation	VLBI frame translation, VLBI frame rotation, 12 site motions. 1×3 + 1×3 + 12×3 = 42 parameters.	VLBI and SLR sites less than 10 km apart move at the same velocity. At 10 places there is 1 SLR and 1 VLBI site. At 2 places there are 1 SLR and 2 VLBI sites.
RAG, 0.80† *Peltier* [1994] 0.91† *Peltier* [1996]	29 VLBI site motions, 14 SLR site motions, minus 3 anomalous SLR radial motions. 29×3 + 14×3 − 3×1 = 126 data.	rotation of North American plate	SLR and VLBI frame translation; SLR and VLBI frame rotation; rotations of Australian, Eurasian, and Pacific plates; 2 components‡ of rotation of African, Antarctic, Nazca, and South American plates. 2×3 + 2×3 + 3×3 + 4×2 = 29 parameters.	Plates move radially and deform laterally as predicted by a rebound model.
RAG=CM, 0.82† *Peltier* [1994] 0.92† *Peltier* [1996]	29 VLBI site motions, 14 SLR site motions, minus 3 anomalous SLR radial motions. 29×3 + 14×3 − 3×1 = 126 data.	SLR frame translation, rotation of North American plate	VLBI frame translation; SLR and VLBI frame rotation; rotations of Australian, Eurasian, and Pacific plates; 2 components‡ of rotation of African, Antarctic, Nazca, and South American plates. 1×3 + 2×3 + 3×3 + 4×2 = 26 parameters.	Plates move radially and deform laterally as predicted by a rebound model. Velocity between rebound-adjusted geocenter and center of mass is negligible over time period of observation.
UG–NUVEL1A, 1.81*	29 VLBI site motions, 14 SLR site motions, minus 3 anomalous SLR radial motions. 29×3 + 14×3 − 3×1 = 126 data.	rotation of African, Antarctic, Australian, Eurasian, Nazca, Pacific North American, and South American plates	SLR and VLBI frame translation, SLR and VLBI frame rotation, 2×3 + 2×3= 12 parameters.	Plates do not move radially and do not deform laterally. Relative plate motion equals that predicted by model NUVEL-1A.

The normalized sample standard deviation (NSSD, which is the square root of reduced chi-square (χ_ν^2) as defined on page 202 of *Bevington* [1969]) is a measure of the misfit of the model to the data.

*NSSD computed without adjusting for postglacial rebound.

†NSSD computed adjusting for the postglacial rebound model of *Peltier* [1994] or that of *Peltier* [1996].

‡The motion of one site on a plate is insufficient information to determine the motion of the plate. Therefore, for the four plates that have just one site, only two of the three components of the plate rotation were estimated. In these instances the horizontal components of site motion are fit exactly; the vertical component of motion contributes to the determination of the motion of the geocenter.

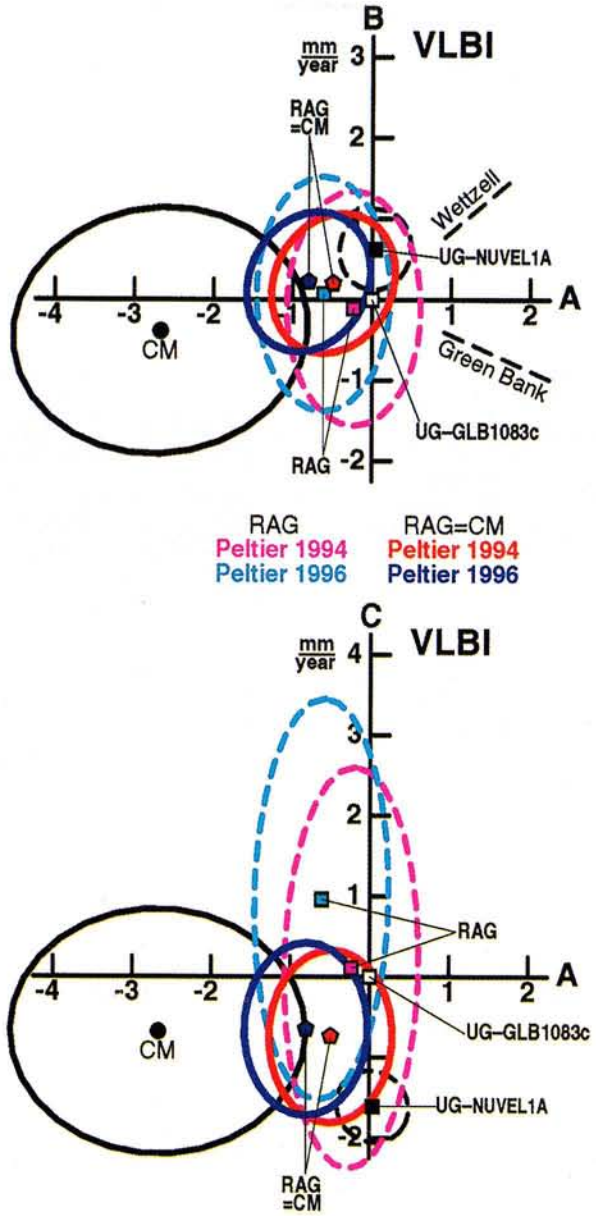
mm/yr, suggesting the Rio Grande rift is currently extending at a very slow rate. The VLBI site at Fort Davis and the SLR site at MacDonald Observatory lie 8 km apart in the Mexican Highland section of the Basin and Range province [*Thelin and Pike*, 1991], an area that may be extending slowly. Therefore we did not assign the two sites to the North American plate. Relative to the North American plate Fort Davis (Texas) is estimated to move toward the northwest at an insignificant rate of 1.1 ±1.2 mm/yr and MacDonald Observatory is estimated to be move toward the northwest at an insignificant rate of 1.4 ±1.6 mm/yr. Fairbanks (Alaska) was not assigned to the North American plate because great M ≥ 7 magnitude earthquakes near Fairbanks suggest that part of Alaska east and north of Fairbanks may be deforming slowly [*Estabrook et al.*, 1988]. Fairbanks is observed to move relative to the North American plate at a significant rate of 3.0 ±1.6 mm/yr, in the opposite direction of that expected from either Pacific-North America plate interaction averaged over many earthquake cycles or interseismic strain buildup due to locking of the main subduction zone at the Aleutian trench. We believe Fairbanks' significant residual motion is a postseismic transient of the great 1964 M=9.2 Prince William Sound earthquake (F. F. Pollitz and D. F. Argus, manuscript in preparation, 1999).

Sites in plate boundary zones may rise or fall due to tectonic processes such as rifting, mountain building, earthquakes, interseismic strain buildup, and postseismic transients. Therefore the motions of sites in plate boundary zones are biased indicators of postglacial rebound and were not used to estimate the motion of the unadjusted geocenter or the rebound-adjusted geocenter.

3.3. Parameters

The reference frame was defined in different ways by varying both the data inverted and the parameters estimated (Table 2). All date correlations were treated in the least squares inversion. In Model RAG, the parameters consist of the angular velocities of the eight major plates, the three rates of rotation and the three rates of translation of the VLBI reference frame, and the three rates of rotation and the three rates of translation of the SLR reference frame. The angular velocity of one plate is fixed; the model is invariant with regard to which plate is fixed. In Model RAG=CM, the constraint that the rebound-adjusted geocenter and the center of mass have the same velocity over the time period of observation was imposed by fixing the three rates of translation of the SLR reference frame to zero.

Sites not on plates were omitted whenever the velocity of the unadjusted geocenter or the rebound-adjusted geocenter was estimated. Introducing the three components of velocity of a site not on a plate would require introducing the three components of the site velocity as estimated parameters; the new parameters



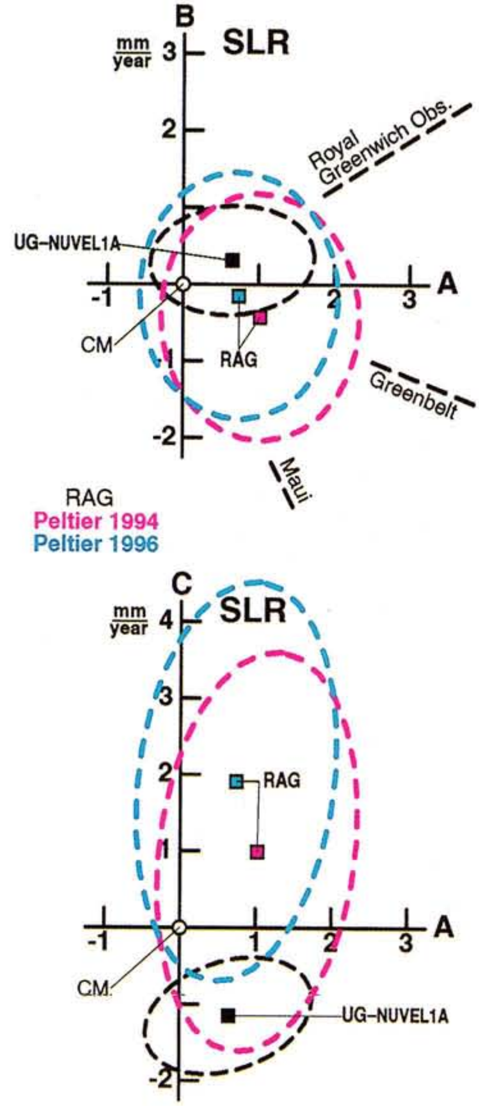


Plate 2. Motion among the points differently defining the three rates of translation of the VLBI reference frame: the rebound-adjusted geocenter (RAG), the center of mass (CM), the unadjusted geocenter (UG–NUVEL1A), and the rebound-adjusted geocenter and center of mass determined assuming that the velocity between the two is negligible over the time period of observation (RAG=CM). Estimates of the motion of the rebound-adjusted geocenter (RAG) and the joint center (RAG=CM) depend on the postglacial rebound model for which adjustments are made and are distinguished by different colors. The coordinate system is defined by the minor (geocentric vector to location A, Plate 1), intermediate (geocentric vector to location B), and major (geocentric vector to location C) axes of the error ellipsoid describing uncertainty in the motion between the rebound-adjusted geocenter and the VLBI network estimated without using SLR's sensitivity to the center of mass. At the origin is UG–GLB1083c, the unadjusted geocenter of C. Ma and J. W. Ryan (electronic communication, 1997).

Plate 3. Motion among the points differently defining the three rates of translation of the SLR reference frame: the rebound-adjusted geocenter (RAG), the center of mass (CM), and the unadjusted geocenter (UG–NUVEL1A). Estimates of the motion of the rebound-adjusted geocenter (RAG) depend on the postglacial rebound model for which adjustments are made and are distinguished by different colors. The coordinate system is the same as in Plate 2. At the origin is the center of mass.

Table 3. Formulas for Additional Systematic Error

	Horizontal	Vertical
VLBI	$\frac{5\text{ mm}}{\text{time yr}}$	$\frac{15\text{ mm}}{\text{time yr}}$
SLR	$\frac{2\text{ mm}}{\sqrt{\text{time yr}}}$	$\frac{5\text{ mm}}{\sqrt{\text{time yr}}}$

The formula by which the systematic error is calculated depends both on whether the datum is from VLBI or SLR and on whether the datum is a horizontal or vertical site velocity component. The formulas are a function of the time period of observation (in years) at a site. The systematic error is added in quadrature to the formal error, that is, the revised variance equals the sum of the square of the formal error plus the square of the systematic error.

would fit the new data exactly, leaving the misfit of the model unchanged. This method provides a means by which we can estimate site velocities relative to plates.

The observed vertical rate of a site on a plate is taken to be the model residual plus the prediction of the rebound model.

3.4. Error Budget

The formal errors in most global geodetic velocity solutions are unrealistically small. For example, data decimation experiments show that site velocities estimated from distinct subsets of VLBI data differ by a factor of 1.5 to 2 times more than predicted from linear propagation of the formal errors [*Ryan et al.*, 1993]. This underestimation of errors is due to apparent seasonal effects; that is, there appear to be correlations among estimates of site position over short time intervals [*Argus and Gordon*, 1996]. Here we did not rescale errors by a multiplicative factor; we instead incorporated a systematic (additive) error [*Argus and Gordon*, 1996]. The formula for this systematic error depends both on whether the datum is from VLBI or SLR and on whether the datum is a vertical or horizontal component of site velocity (Table 3). For the horizontal component of a VLBI site velocity, we incorporated a systematic error consisting of a distance of 5 mm divided by the number of years of observations at a given site. For example, to incorporate an additional error of 5 mm for a site with 10 years of observation, we added the square of 0.5 mm/yr (= 5 mm/10 years) to the square of the 1σ error in each velocity component to obtain the revised variance. The size of the horizontal systematic error was calibrated (Table 4) by finding the value that gives a normalized sample standard deviation of ~1 if the plates were deforming laterally as predicted by the two postglacial rebound models. The size of the vertical systematic error was calibrated (Table 4) by finding the value that gives a normalized sample standard deviation of ~1 if the plates were moving radially as predicted by the two models.

3.5. Data Importances

Data importances (Table 1), which measure the information content of each velocity component input, were computed using equation (19) of *Minster et al.* [1974]. The importance of a datum measures what fraction of a parameter the datum is constraining; the data importances sum to the number of estimated parameters.

4. Reference Frame Results

Motion among the points differently defining the three rates of translation of the VLBI reference frame and the three rates of translation of the SLR reference frame are plotted in Plate 2 and in Plate 3, respectively. The motions are plotted in the coordinate system defined by the minor (geocentric vector to location A, Plate 1), intermediate (geocentric vector to location B), and major (geocentric vector to location C) axes of the error ellipsoid describing uncertainty in the motion between the rebound-adjusted geocenter and the VLBI network estimated without using SLR's sensitivity to the center of mass. The motion of the geocenter relative to the VLBI network is best constrained in the A-B plane, which intersects the Earth's surface in Europe and eastern North America, the two places with the best constrained VLBI radial motions. Plotting the motions in this special coordinate system allows reference frame differences to be assessed in the directions that matter the most for the interpretation in this paper.

4.1. Rebound-Adjusted Geocenter (RAG)

Assuming that the plates move radially and deform laterally as predicted by a postglacial rebound model constrains the VLBI reference frame moderately well. The motion of the rebound-adjusted geocenter (Plate 2, RAG) relative to the VLBI network is constrained well in the A direction, moderately well in the B direction, and poorly in the C direction. The 95% confidence ellipsoid describing uncertainty in this motion has principal semi-axes of 1.0 mm/yr, 1.6 mm/yr, and 2.9 mm/yr. The geocenter adjusted for the model of *Peltier* [1994] (magenta RAG) moves at 0.9 mm/yr (0.4 mm/yr in the A direction) relative to the geocenter adjusted for the model of *Peltier* [1996] (light blue RAG). This relative motion results in considerable differences in estimated VLBI radial site motions.

The assumption constrains the SLR reference frame poorly but nevertheless provides an estimate of the motion of the rebound-adjusted geocenter (Plate 3, RAG) relative to the center of mass (CM) as observed by SLR. The geocenter adjusted for the model of *Peltier* [1994] (magenta RAG) moves relative to the center of mass at an insignificant 1.5 ±1.7 mm/yr toward the location 70°N, 161°W. The geocenter adjusted for the model of *Peltier* [1996] (light blue RAG) moves relative to the center of mass at an insignificant 2.0 ±2.1 mm/yr toward the location 58°N, 139°E. (Motion is considered insignificant if the three-dimensional 95% confidence limits include zero.) The insignificant observed motions are consistent with the hypothesis that the velocity between rebound-adjusted geocenter and the center of mass is negligible over the time period of observation.

Caution need be taken when assessing an estimate of the motion between the rebound-adjusted geocenter and the center of mass because the estimated speed is always biased upward, away from zero. If the true speed were zero, the estimated speed would

Table 4. Normalized Sample Standard Deviation for Different Data Types

PGR Model Adjusted For	VLBI Horizontal	VLBI Vertical	SLR Horizontal	SLR Vertical
Peltier [1994]	0.80	0.90	0.72	0.92
Peltier [1996]	0.99	0.96	0.77	0.85

The normalized sample standard deviation (NSSD, the square root of reduced chi-square (χ^2_ν) as defined on page 202 of *Bevington* [1969]) was computed using the usual formula except that the importance of the data subset was substituted for the number of parameters. The NSSD's are for the model in which the velocity between the rebound-adjusted geocenter and the center of mass is assumed to be negligible over the time period of observation. Insofar as the normalized sample standard deviations are slightly less than 1, the uncertainties are slightly conservative.

always be greater than zero. Monte Carlo simulation was performed to determine unbiased estimates of speed and unbiased one-sided upper 95% confidence bounds. Unbiased speeds and unbiased upper bounds were defined in the same way as *Argus and Gordon* [1996] defined them for how fast a site may move relative to the plate on which it lies. (We expanded their formulation from two to three dimensions.) The unbiased estimate of the speed of motion between the center of mass and the geocenter adjusted for the model of *Peltier* [1994] was found to be –0.5 mm/yr and the upper 95% confidence bound on this speed was found to be 2.9 mm/yr. (A negative unbiased speed indicates that a hypothetical true speed of zero gives speeds greater than the estimated apparent speed in over 50% of the realizations.) The unbiased estimate of the speed of motion between the center of mass and the geocenter adjusted for the model of *Peltier* [1996] was found to be 0.6 mm/yr and the upper 95% confidence bound on this speed was found to be 3.8 mm/yr. Insofar as the two postglacial rebound models represent the range of possible intraplate deformation, the geodetic data limit the motion of the rebound-adjusted geocenter and the center of mass over the time period of observation to be < 4 mm/yr.

4.2. Center of Mass (CM)

Assuming only that VLBI and SLR sites less than 10 km apart move at the same velocity constrains the VLBI reference frame poorly. The motion of the center of mass (Plate 2, CM) relative to the VLBI network is highly uncertain even in the A direction. Therefore using only SLR's sensitivity to the center of mass without assigning sites to plates leaves the VLBI reference frame too poorly constrained to be useful for the interpretation of VLBI radial site motions in Europe and eastern North America.

4.3. Rebound-Adjusted Geocenter Equals Center of Mass (RAG=CM)

Assuming that the plates move radially and deform laterally as predicted by a postglacial rebound model and that the velocity between the rebound-adjusted geocenter and the center of mass is negligible over the time period of observation constrains the VLBI reference frame very well. The motion of this joint center (Plate 2, RAG=CM) relative to the VLBI network has a 95% confidence ellipsoid with principal semi-axes of 0.8 mm/yr, 1.0 mm/yr, and 1.2 mm/yr. The joint center adjusted for the model of *Peltier* [1994] (red RAG=CM) moves at 0.3 mm/yr (nearly all in the A direction) relative to the joint center adjusted for the model of *Peltier* [1996] (navy blue RAG=CM). The two joint centers move at 0.8–1.1 mm/yr (0.5–0.8 mm/yr in the negative A direction) relative to the unadjusted geocenter (UG–GLB1083) of C. Ma and J. W. Ryan (electronic communication, 1997), which results in big differences in estimated VLBI radial site motions.

4.4. Unadjusted Geocenter Assuming a Plate Motion Model (UG–NUVEL1A)

Assuming both that the plates do not move radially and that they move laterally as predicted by plate motion model NUVEL-1A constrains the VLBI reference frame extremely well. The motion of the unadjusted geocenter (Plate 2, UG–NUVEL1A) relative to the VLBI network is constrained very tightly. However, the assumption increases data misfits (Table 2) by a factor of ~2. Therefore the estimated motion of the unadjusted geocenter relative to the VLBI network is probably biased by either errors in NUVEL-1A or differences between plate motions over the past 20 years and those over the past 3 Myr. It would be unwise to accept the apparent result that the unadjusted geocenter (Plate 3, UG–NUVEL1A) is moving at a significant velocity relative to the center of mass (CM).

5. Postglacial Rebound Results

5.1. Vertical Motion

Observed vertical motions are compared with rebound model predictions in Figure 5. The observed vertical motions are relative to both the rebound-adjusted geocenter and the center of mass and are determined assuming that the velocity between the two centers is negligible over the time period of observation. The observed vertical motion of VLBI sites depends on the rebound model for which adjustments were made, whereas the observed vertical motion of SLR sites does not. The VLBI vertical estimates determined adjusting for the model of *Peltier* [1996] (Figure 5, right-hand side) have 0.2–0.3 mm/yr more uplift in Europe and eastern North American than those determined adjusting for the model of *Peltier* [1994] (Figure 5, left-hand side).

There is a strong correlation between observed and predicted vertical motions at sites with a long time period of observation. The overall misfit, however, is significantly greater than expected from the formal errors. In the comparison against the model of *Peltier* [1994], 11 of the 16 estimated vertical motions of sites on plates with data over 8 years or longer are misfit by more than 1σ and 7 are misfit by more than 2σ. The weighted root mean square of residuals is 1.1 mm/yr in both the comparison against the model of *Peltier* [1994] and that against the model of *Peltier* [1996].

Observed and predicted vertical motions along profiles across Laurentia and Fennoscandia are shown in Figure 6. The observed vertical site motion is taken to be the mean of the estimate determined adjusting for the model of *Peltier* [1994] and that determined adjusting for the model of *Peltier* [1996]. Realistic 95% confidence limits determined using the modified error budget range from ±2 mm/yr to ±3 mm/yr. (Hereinafter the values quoted after the "±" are 95% confidence limits.) However, the uncertainty in the relative vertical motion of nearby VLBI sites is less because the relative uncertainty is not subject to uncertainty in the reference frame. The observed vertical motions of sites in Europe and eastern North America that we estimate have 0.4–0.6 mm/yr more uplift than those estimated by C. Ma. and J. W. Ryan (electronic communication, 1997).

The lateral gradient in vertical site motion going away from the Laurentide ice sheet center is constrained well in eastern North America. The site at Algonquin Park (Ontario) is observed to be rising at 2 mm/yr, reflecting unloading of the Laurentide ice sheet. (Hereinafter vertical site motions are quoted to the nearest half mm/yr.) The forebulge produced by the Laurentide ice sheet is observed to be collapsing slowly: the observed vertical motion of Haystack (Massachusetts) is 0 mm/yr, Westford (Massachusetts) and Greenbelt (Maryland) are both subsiding at 1 mm/yr, and Green Bank (West Virginia) is subsiding at 1.5 mm/yr. The slow observed subsidence of the east coast of the United States favors the model of *Peltier* [1996] over that of *Peltier* [1994]. Richmond (Florida) is subsiding at 0.5 mm/yr, which is consistent with the minor motion expected far from the ice sheets.

The VLBI site at Yellowknife (Northwest Territories) is observed to be rising fast, at 8 ±5.5 mm/yr, although this vertical motion is highly uncertain because it is estimated from only 6

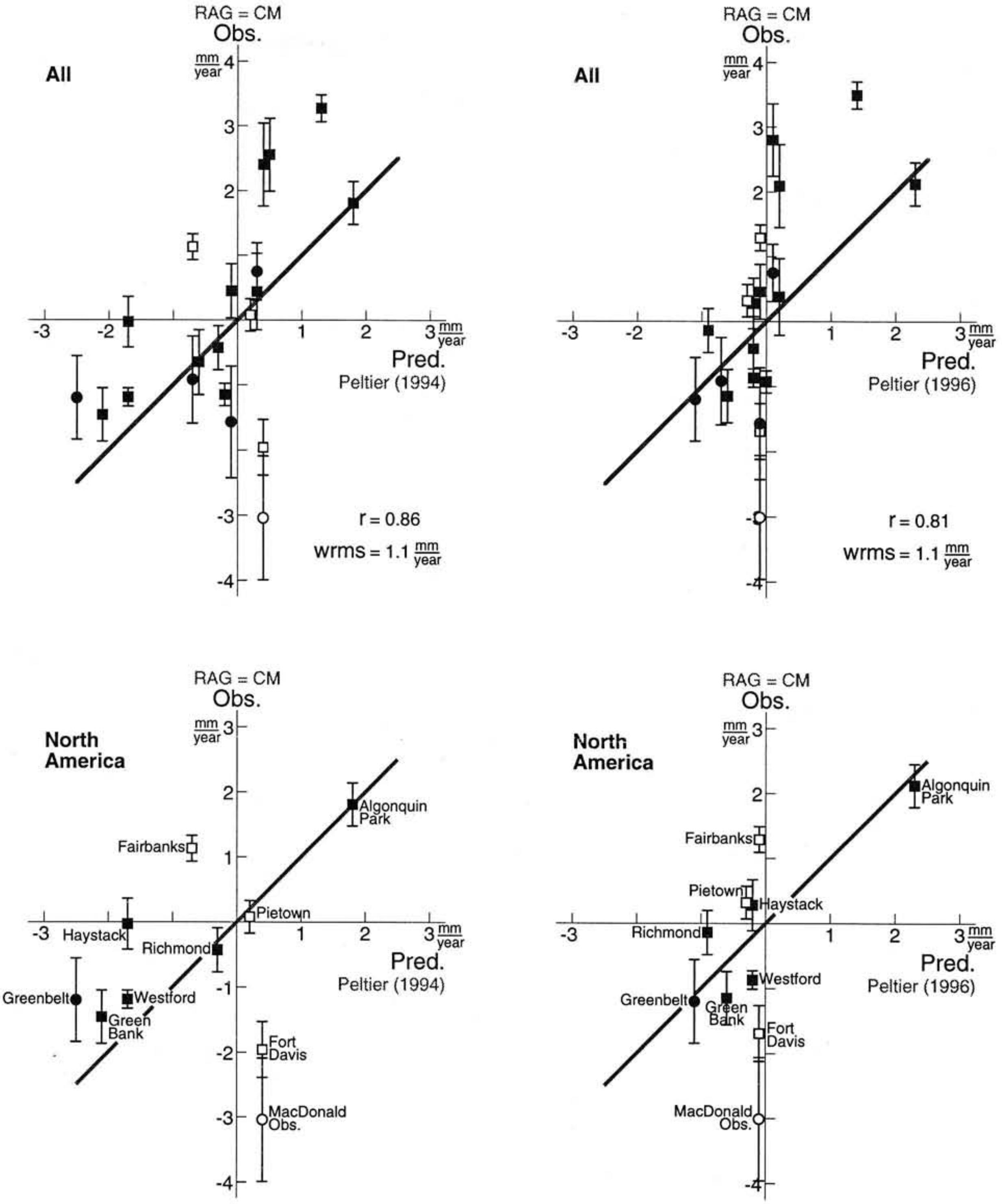


Figure 5. Rebound predictions versus observed vertical site motions relative to the rebound-adjusted geocenter and the center of mass estimated assuming that the velocity between the two is negligible over the time period of observation. The observed vertical motion of VLBI sites (squares) depends on the rebound model for which adjustments were made, whereas the observed vertical motion of SLR sites (circles) does not. In the four plots on the left, the predictions of the model of *Peltier* [1994] are compared with the estimates determined adjusting for that model. In the four plots on the right, the predictions of the model of *Peltier* [1996] are compared with the estimates determined adjusting for that model. The top two plots show the vertical motions of all sites with data over 8 years or longer. The bottom six plots show the same vertical site motions segregated by plate. Error bars are the formal standard (1σ) errors in solutions GLB1083c (C. Ma and J. W. Ryan, electronic communication, 1997) and CSR96L01 (R. J. Eanes and M. M. Watkins, electronic communication, 1996). The correlation coefficient (r) and weighted root mean square (wrms) were calculated for sites on plate interiors (solid symbols) but not for sites along plate margins (open symbols).

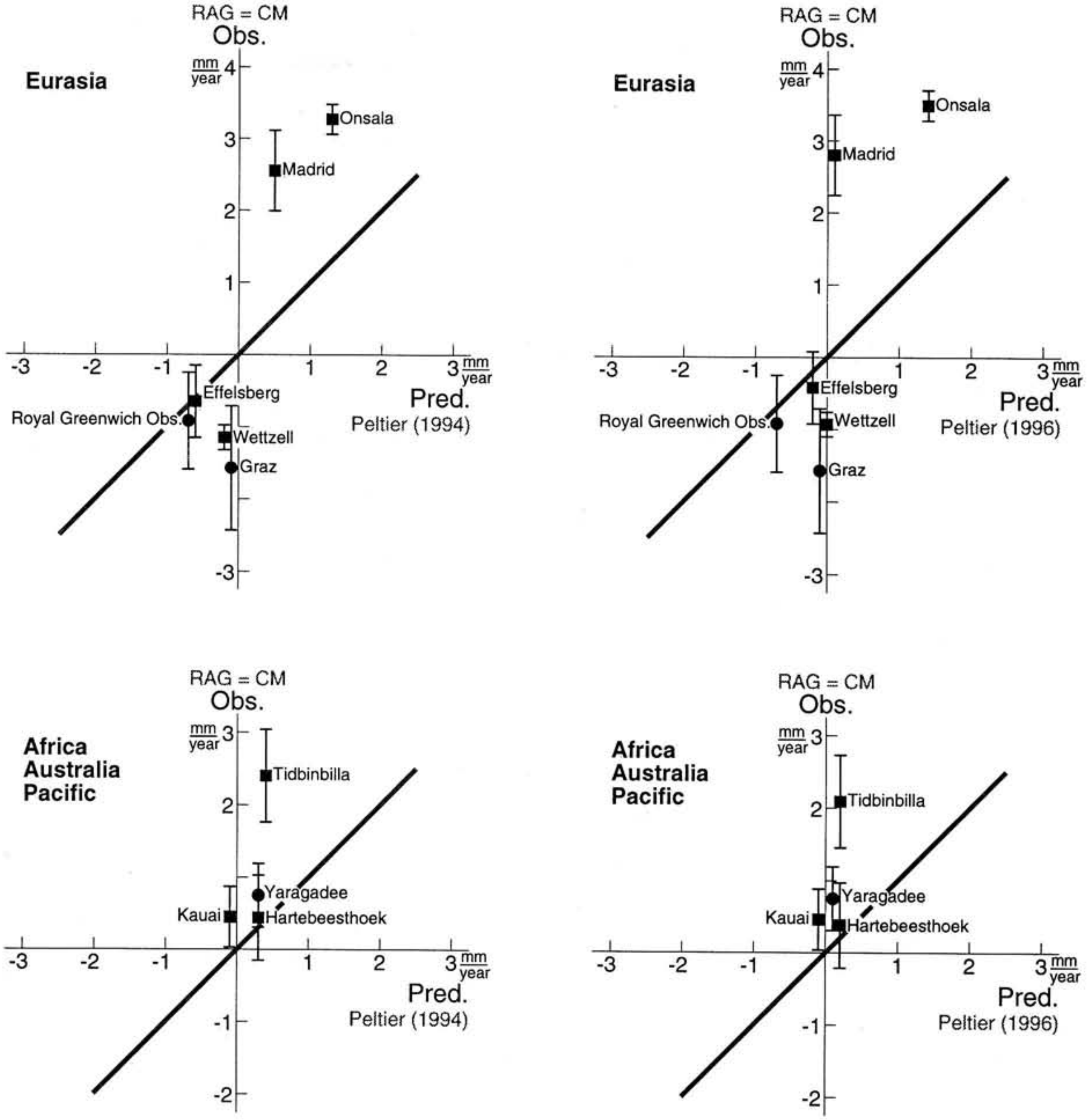


Figure 5. (continued)

years of data. Independent results from Global Positioning System (GPS) geodesy (M. B. Heflin, electronic communication, 1999) support this fast rise: the permanent GPS site at Yellowknife is estimated to be rising at 8 ±4 mm/yr using data taken from 1991 to 1999. Yellowknife's fast observed rise suggests that the western Laurentide ice sheet was thicker during the last glacial maximum than that in the models of *Peltier* [1994] and *Peltier* [1996]. Fairbanks (Alaska) is observed to be rising at 1 mm/yr, which is consistent with the minor subsidence predicted by the two postglacial rebond models. Effects associated with Pacific-North America plate interaction, especially a postseismic transient due to the 1964 M=9.2 Prince William Sound earthquake (see Section 3.2), probably influence Fairbanks' vertical motion.

The VLBI site at Fort Davis (Texas) and the SLR site at MacDonald Observatory (Texas) are subsiding anomalously fast, at 2 mm/yr and 3 mm/yr, respectively, which is inconsistent with the vanishing vertical motion expected far from the ice sheets. The two sites lie 8 km apart, ruling out the possibility of an effect local to either site. The insignificant horizontal motion (quoted in Section 3.2.) of Fort Davis and MacDonald Observatory relative to the North American plate suggests that normal faulting associated with the Rio Grande rift does not produce the observed subsidence.

The lateral gradient in vertical site motion going away from the Fennoscandian ice sheet center is constrained moderately well in Europe. Onsala (Sweden) is observed to be rising at 3.5 mm/yr, reflecting unloading of the Fennoscandian ice sheet. Onsala's observed uplift is significantly faster than predicted by both the model of *Peltier* [1994] and that of *Peltier* [1996], suggesting that the solid Earth is farther from its equilibrium state (that is, the state of the solid Earth undeformed by loading of the ice sheets) than in both models. Greater disequilibrium would result from either a thicker Fennoscandian ice sheet during the

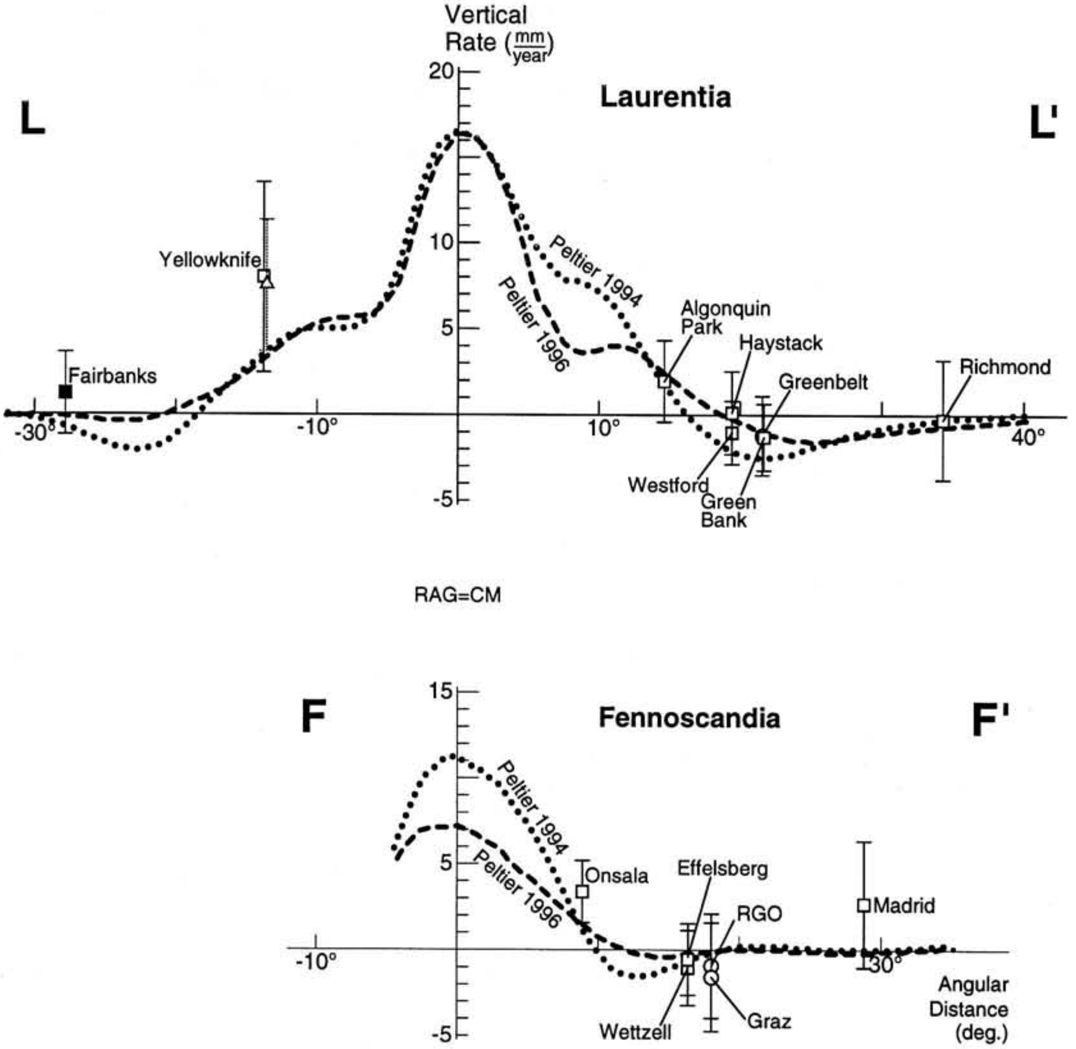


Figure 6. Rebound predictions and observed vertical motions versus distance along profiles across Laurentia (L-L') and Fennoscandia (F-F'). Profile locations are shown in Figure 1. Observed vertical motions are relative to the rebound-adjusted geocenter and the center of mass determined assuming that the velocity between the two is negligible over the time period of observation. The observed vertical motion of a VLBI (square) site is taken to be the mean of that estimated adjusting for the model of *Peltier* [1994] and that estimated adjusting for the model of *Peltier* [1996]. The observed vertical motion of a SLR (circle) site is independent of the postglacial rebound model for which adjustments were made. Error bars are 95% confidence limits determined from the realistic error budget. Vertical motions are plotted for all sites on plates with data over 8 years or longer; the vertical motion of the VLBI site at Yellowknife, which has data over 6 years, is also plotted. The vertical motion of the GPS site (triangle) at Yellowknife, which has data over 8 years, is from results by M. B. Heflin (electronic communication, 1999).

last glacial maximum or a more viscous upper mantle. If the upper mantle were more viscous, the uplift rate would have decreased more slowly with time, resulting in a faster rate today.

Sites in Europe farther from the Fennoscandian ice sheet center are observed to subside slowly: Effelsberg (Germany), Wettzell (Germany), Graz (Austria), and the Royal Greenwich Observatory (England) are subsiding at 0.5–1.5 mm/yr, perhaps reflecting the slow collapse of the forebulge produced by the ice sheet. Madrid (Spain) is observed to be rising at 2.5 mm/yr, insignificantly faster than in both rebound models.

5.2. Horizontal Motion

The areas beneath the margins of the Laurentide ice sheet are moving laterally outward away from the ice sheet center moderately fast in the model of *Peltier* [1996], whereas these areas are moving outward slowly in the model of *Peltier* [1994] (Figure 1). The VLBI sites at Algonquin Park and Yellowknife are moving apart at 3.6 mm/yr in the model of *Peltier* [1996] and at 1.2 mm/yr in the model of *Peltier* [1994]. The observed separation at 0.6 ±2.0 mm/yr is consistent with the model of *Peltier* [1994] but inconsistent with the model of *Peltier* [1996].

Except at Yellowknife, the horizontal predictions of the models of *Peltier* [1994] and *Peltier* [1996] are difficult to distinguish. The predicted horizontal site motions differ significantly between the two models, but the predicted intraplate deformation does not, and the data are only sensitive to the latter. For example, Algonquin Park and Richmond are converging at 1.6 mm/yr in the model of *Peltier* [1996] and at 0.9 mm/yr in the model of *Peltier* [1994]. The observed convergence at 1.1 ±1.4 mm/yr is consistent with either model.

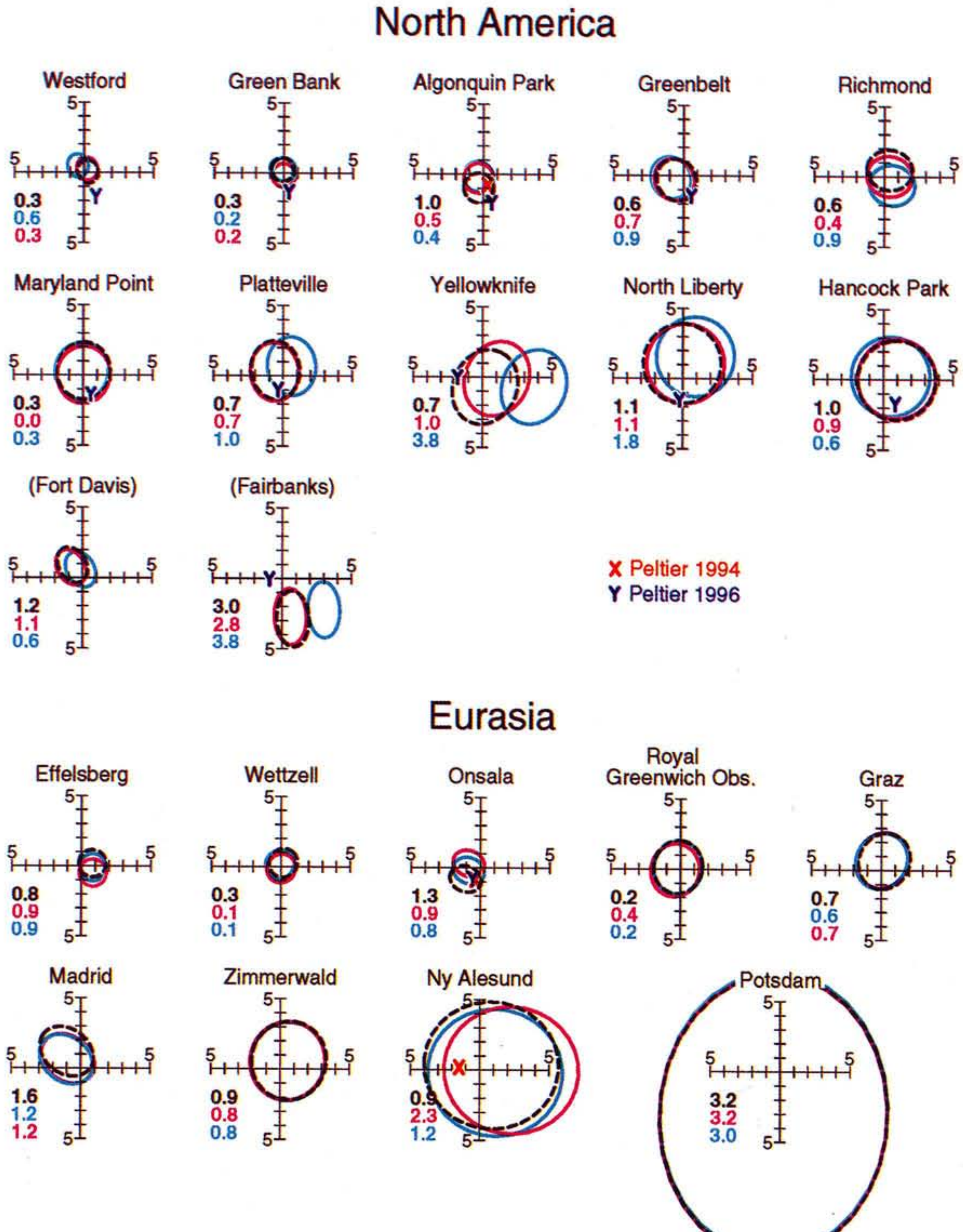


Plate 4. Estimates of motions of sites relative to their home plates calculated without adjusting for postglacial rebound (black dashed 95% confidence ellipses), calculated adjusting for the model of *Peltier* [1994] (magenta 95% confidence ellipses), and calculated adjusting for the model of *Peltier* [1996] (light blue 95% confidence ellipses). The predictions of the models of *Peltier* [1994] (red X's) and *Peltier* [1996] (navy blue Y's) are shown where they are ≥ 0.5 mm/yr. Fairbanks (Alaska) and Fort Davis (Texas) were not assumed to be on the North American plate.

Intraplate deformation is assessed more completely in Plate 4, which shows the 95% confidence limits (black dashed ellipse) in the horizontal motion of each site on a plate relative to its home plate. The observations indicate that the plate interiors are rigid to a high degree. Of the 19 sites on the Eurasian and North American plates, 17 have residual motions < 1.5 mm/yr. The residual motion of every site but Onsala is insignificant, that is, the 95% confidence limits include the origin. Onsala is observed to move toward the southwest at a significant 1.3 ±0.8 mm/yr, away from the center of the Fennoscandian ice sheet. This observation is consistent with both the prediction of the model of *Peltier* [1994] and that of *Peltier* [1996]. The residual motions are near the 95% confidence limits at three more sites: Algonquin Park, Effelsberg, and Madrid. Algonquin Park is observed to move toward the south at 1.0 ±0.8 mm/yr, away from the center of the Laurentide ice sheet, which is consistent with the prediction of the model of *Peltier* [1994] but inconsistent with the prediction of the model of *Peltier* [1996]. Madrid is estimated to move toward the northwest at 1.6 ±1.6 mm/yr relative to the Eurasian plate, in the same direction as the motion of the African plate, suggesting that Africa-Eurasia plate interaction influences Madrid's motion.

The data provide tight upper bounds on how fast a site may be moving relative to its home plate (Figure 7). Speeds greater than 2.0 mm/yr are excluded for nine of the 19 plate-interior sites, consisting of five of the 10 North American plate sites and four of the nine Eurasian plate sites. Two more North American plate sites and one more Eurasian plate site have upper bounds near this limit. These upper bounds are smaller than those of *Argus and Gordon* [1996] and *Dixon* [1996].

Comparisons between predicted and observed residual site motions can be misleading because glacial isostatic adjustment may bias not only the estimate of the motion of a site, but also the estimate of the motion of a plate. A postglacial rebound model can be better assessed by first adjusting for the motion predicted by the model and next estimating the motion of each site relative to its home plate. Adjusting for the model of *Peltier* [1994] (Plate 4, magenta 95% confidence ellipses) reduces the misfit of the rigid plate model: the residual site motions are reduced at Algonquin Park and Onsala and are altered little elsewhere. Adjusting for the model of *Peltier* [1996] (Plate 4, light blue 95% confidence ellipses) increases the misfit of the rigid plate model: Yellowknife's residual motion becomes significant and large, reflecting the observation that Yellowknife is not moving laterally away from eastern North America as predicted by *Peltier* [1996]. When adjustments are made for the model of *Peltier* [1996], the motion of the North American plate in eastern North America becomes more northwestly to compensate for the southeastward

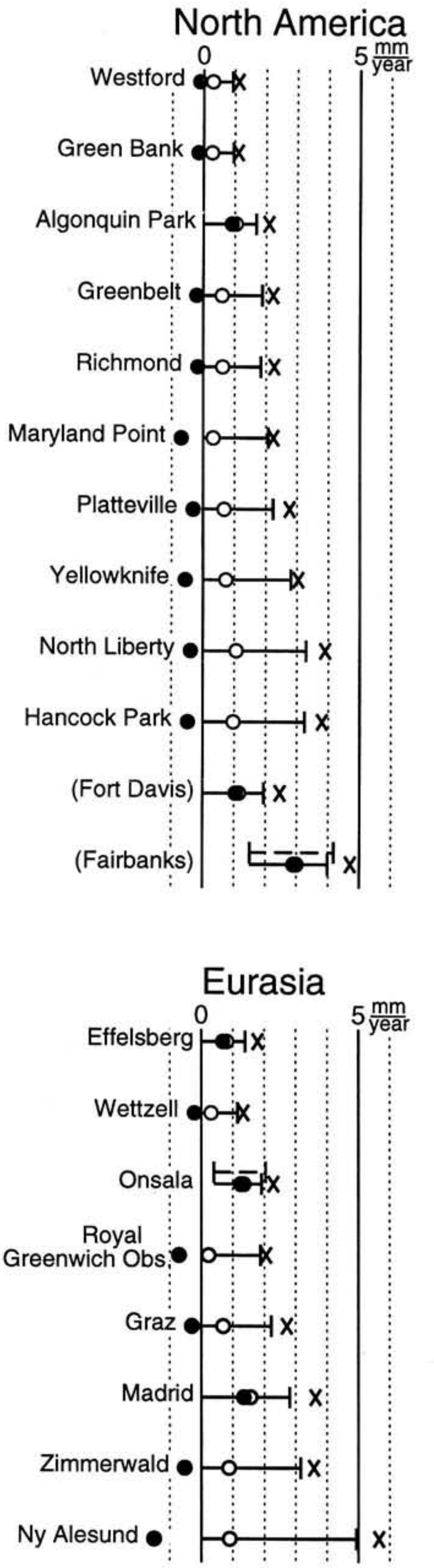


Figure 7. Speeds and confidence limits between sites and their home plates. The apparent speed (open circle) estimated from the magnitude of the velocity (in Plate 4) between the site and its home plate is biased upward away from zero, as is the upper 95% confidence limit (X) in this speed taken to be the farthest point along the 95% confidence limits (in Plate 4). Monte Carlo simulation was used to determine an unbiased estimate of the true speed between a site and its home plate and the one-sided 95% confidence upper limit in this speed following the method of *Argus and Gordon* [1996]. The unbiased estimate (solid circle) of the true speed is the hypothetical true speed that gives an expected speed equal to the observed apparent speed. In the instances in which no value of the hypothetical true speed is small enough to give an expected speed equal to that observed, the expected value for the case of zero true speed is subtracted from the observed speed to give the unbiased estimate of the true speed. The one-sided upper 95% confidence limit (vertical line at right-hand side of error bar) is the hypothetical true speed for which only 5% of the realizations are less than the observed apparent speed. The two-sided 95% confidence limits (dashed) are shown for Fairbanks and Onsala, the two sites for which these two-sided limits exclude zero; the lower two-sided 95% confidence limit is the hypothetical true speed for which 2.5% of the realizations are greater than the observed apparent speed.

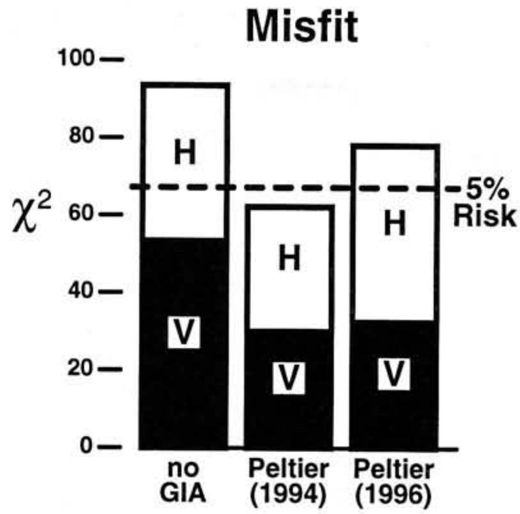


Figure 8. The total, horizontal (H), and vertical (V) misfits of a model with no glacial isostatic adjustment (no GIA) and the postglacial rebound models of *Peltier* [1994] and *Peltier* [1996]. The 5% risk level at which a reduction in misfit by a postglacial rebound model is significant is shown.

motion generated by glacial isostatic adjustment in the model, resulting in the poor fit at Yellowknife.

5.3. Goodness of Fit

Prior to adjusting for rebound, the rigid plate model most poorly fits the site motions where glacial isostatic adjustment is expected to be fastest: misfits are largest at Onsala ($\chi^2 = 26.7$), Algonquin Park ($\chi^2 = 9.4$), and Yellowknife ($\chi^2 = 9.4$). Adjusting for the model of *Peltier* [1994] reduces the misfit at each of these three sites by a factor of 3; the total misfit is reduced by 34%, which is significant at the 1.8% risk level (Figure 8). Adjusting for the model of *Peltier* [1996] reduces the misfit at Onsala and Algonquin Park by a factor of 3 and the misfit at Yellowknife by a factor of 2; the total misfit is reduced by 16%, which is significant at the 19% risk level. The observed horizontal site motions are fit significantly better by the model of *Peltier* [1994] than by that of *Peltier* [1996]; the observed vertical site motions are fit about equally well by the two models (Figure 8). Adjusting for the model of *Peltier* [1994] reduces the vertical misfit by 44% and the horizontal misfit by 18%. Adjusting for the model of *Peltier* [1996] reduces the vertical misfit by 39% but increases the horizontal misfit by 16%.

6. Conclusions

1. The common practice of estimating the motion of the geocenter assuming both that sites do not move radially and that sites move laterally as predicted by NUVEL-1A may result in biases in comparisons between observed site motions and postglacial rebound models predictions. Site motions associated with plate motion and those due to glacial isostatic adjustment are best distinguished by estimating the plate motion using only the geodetic data and attributing intraplate deformation to glacial isostatic adjustment.

2. The geodetic estimate of the velocity between the rebound-adjusted geocenter and the center of mass averaged over the time period of observation differs insignificantly from zero. The geodetic data place an upper bound of 4 mm/yr on this relative velocity.

3. The observed vertical site motions are consistent with the postglacial rebound models of *Peltier* [1994] and *Peltier* [1996] excepting the observed anomalous subsidence of two sites in Texas. Relative sea level histories over the Holocene observed along the northeast coast of the United States [*Peltier*, 1998], however, agree with the very slow collapse of the Laurentide forebulge predicted by the model of *Peltier* [1996] but rule out the more rapid collapse predicted by the model of *Peltier* [1994]. The long history of VLBI and SLR observation provides high-quality estimates of radial site motions relative to the rebound-adjusted geocenter and the center of mass assuming that the velocity between the two is negligible over the time period of observation. The determination of the reference frame using VLBI and SLR is important for Global Positioning System (GPS) studies of glacial isostatic adjustment in Laurentia, Fennoscandia [*BIFROST*, 1996], Greenland, and Antarctica.

4. The areas beneath the margins of the late Pleistocene Laurentide and Fennoscandian ice sheets are moving laterally outward from the ice sheet centers very slowly if at all, in disagreement with the moderately fast outward motion predicted by the model of *Peltier* [1996]. This observation provides an important new constraint that should be satisfied by future postglacial rebound models.

5. The apparent discrepancy between (1) the preference of the slow lateral outward observed motions for the model of *Peltier* [1994] and (2) the preference of the very slow observed Holocene subsidence of the northeastern United States for the model of *Peltier* [1996] suggests that the mantle viscosity must be further modified. We are investigating the consequences of viscosity refinements of a radially-invariant mantle; we are also studying the possible resolution of the discrepancy by the introduction of lateral variations in mantle viscosity.

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References

Argus, D. F., Postglacial rebound from VLBI geodesy: On establishing vertical reference, *Geophys. Res. Lett., 23,* 973–976, 1996.

Argus, D. F., and R. G. Gordon, No-net-rotation model of current plate velocities incorporating plate motion model NUVEL-1, *Geophys. Res. Lett., 18,* 2038–2042, 1991.

Argus, D. F., and R. G. Gordon, Tests of the rigid-plate hypothesis and bounds on intraplate deformation using geodetic data from very long baseline interferometry, *J. Geophys. Res., 101,* 13,555–13,572, 1996.

Bevington, P. R., *Data Reduction and Error Analysis for the Physical Sciences,* McGraw–Hill, New York, 1969.

BIFROST project members, GPS measurements to constrain geodynamic processes in Fennoscandia, *Eos Trans. AGU, 77,* 337–341, 1996.

Boucher, C., Z. Altamimi, M. Feissel, and P. Sillard, Results and analysis of the ITRF94, *IERS Tech. Note 20,* Int. Earth Rotation Serv., Paris, 1996.

Boucher, C., Z. Altamimi, and P. Sillard, Results and analysis of the ITRF96, *IERS Tech. Note 24,* Int. Earth Rotation Serv., Paris, 1998.

DeMets, C., R. G. Gordon, D. F. Argus, and S. Stein, Current plate motions, *Geophys. J. Int., 101,* 425–478, 1990.

DeMets, C., R. G. Gordon, D. F. Argus, and S. Stein, Effect of recent revisions to the geomagnetic reversal time scale on estimates of current plate motion, *Geophys. Res. Lett., 21,* 2191–2194, 1994.

Dixon, T. H., A. Mao, and S. Stein, How rigid is the stable interior of the North American plate, *Geophys. Res. Lett., 23,* 3035–3038, 1996.

Dong, D., J. O. Dickey, Y. Chao, and M. K. Cheng, Geocenter variations caused by atmosphere, ocean, and surface ground water, *Geophys. Res. Lett., 24,* 1867–1870, 1997.

Dziewonski, A. M., and D. L. Anderson, Preliminary reference Earth model, *Phys. Earth Planet. Inter., 25,* 297–356, 1981.

Estabrook, C. H., D. B. Stone, and J. N. Davies, Seismotectonics of northern Alaska, *J. Geophys. Res., 93,* 12,026–12,040, 1988.

Forte, A. M., and J. X. Mitrovica, New inferences of mantle viscosity from joint inversion of long-wavelength mantle convection and postglacial rebound data, *Geophys. Res. Lett., 23,* 1147–1150, 1996.

Heki, K., Horizontal and vertical crustal movements from three-dimensional very long baseline interferometry kinematic reference frame: Implication for the reversal time scale revision, *J. Geophys. Res., 101,* 3187–3198, 1996.

Kar, S., Long-period variations in the geocenter observed from laser tracking of multiple earth satellites, Ph.D. thesis, Univ. of Tex. at Austin, 1997.

Lambeck, K. P., P. Johnston, C. Smither, and M. Nakada, Holocene glacial rebound and sea-level change in northwest Europe, *Geophys. J. Int., 103,* 451–468, 1990.

Ma, C., J. M. Gipson, D. Gordon, D. S. Caprette, and J. W. Ryan, Site positions and velocities, earth orientation parameters, and source positions from the NASA space geodesy program-GSFC using linear velocity for HRAS 085: Solution GLB886a, *IERS Tech. Note 14,* R-19, Int. Earth Rotation Serv., Paris, 1993.

Ma, C., J. W. Ryan, and D. S. Caprette, NASA space geodesy program–GSFC data analysis–1993: VLBI geodetic results 1979-92, *NASA Tech. Memo, TM-104605,* 231 pp., 1994.

McConnell, R. K., Viscosity of the mantle from relaxation time spectra of isostatic adjustment, *J. Geophys. Res., 73,* 7089–7105, 1968.

Minster, J. B., T. H. Jordan, P. Molnar, and E. Haines, Numerical modeling of instantaneous plate tectonics, *Geophys. J. R. Astron. Soc., 36,* 541–576, 1974.

Pavlis, N. K., and R. H. Rapp, The development of an isostatic gravitational model to degree 360 and its use in global gravity modelling, *Geophys. J. Int., 100,* 369–378, 1990.

Peltier, W. R., Ice age paleotopography, *Science, 265,* 195–201, 1994.

Peltier, W. R., Mantle viscosity and ice-age ice sheet topography, *Science, 273,* 1359–1364, 1996.

Peltier, W. R., A space geodetic target for mantle viscosity discrimination–horizontal motions induced by glacial isostatic adjustment, *Geophys. Res. Lett., 25,* 543–546, 1997.

Peltier, W. R., Postglacial variations in the level of the sea: Implications for climate dynamics and solid earth geophysics, *Rev. Geophys., 36,* 603–689, 1998.

Peltier, W. R., and X. Jiang, Mantle viscosity from the simultaneous inversion of multiple data sets pertaining to postglacial rebound, *J. Geophys. Res., 23,* 503–506, 1996.

Pirazzoli, P. A., *World Atlas of Holocene Sea-Level Changes, Elsevier Oceanogr. Ser., vol. 58,* Elsevier, New York, 1991.

Ryan, J., T. Clark, C. Ma, D. Gordon, D. S. Caprette, and W. E. Himwich, Global scale plate motions measured with CDP VLBI data, in *Space Geodesy and Geodynamics, Geodyn. Ser.,* vol. 23, edited by D. E. Smith and D. L. Turcotte, pp. 37–49, AGU, Washington, D. C., 1993.

Simons, M., and B. H. Hager, Localization of the gravity field and the signature of glacial rebound, *Nature, 390,* 500–504, 1997.

Stephenson, E. R., and L. V. Morrison, Long term fluctuations in the Earth's rotation: 700 B.C. to A.D. 1990, *Philos. Trans. R. Soc. London, Ser. A,* 165–202, 1995.

Thelin, G. P., and R. J. Pike, Landforms of the conterminous United States–A digital shaded-relief portrayal, *U.S. Geol. Surv. Map, I-2206,* 1991.

Vicente, R. O., and S. Yumi, Co-ordinates of the pole (1899-1968) returned to the conventional international origin, *Publ. Int. Latitude Observ. Mizusawa, 7,* 41–50, 1969.

Watkins, M. M., and R. J. Eanes, Long-term changes in the earth's shape, rotation, and geocenter, *Adv. Space Res., 13,* 251–255, 1993.

Watkins, M. M., and R. J. Eanes, Observations of tidally coherent diurnal and semidiurnal variations in the geocenter, *Geophys. Res. Lett., 24,* 2231–2234, 1997.

Watkins, M. M., R. J. Eanes, and C. Ma, Comparison of terrestrial reference frame velocities determined from SLR and VLBI, *Geophys. Res. Lett., 21,* 169–172, 1994.

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